MA 182 **SUMMARY – INFINITE SERIES**

nth-TERM TEST

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ may converge or it may diverge.

GEOMETRIC SERIES

The series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots + ar^{n-1} + \cdots$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

p-SERIES

The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if p>1 and diverges if $p \leq 1$.

INTEGRAL TEST

If f is continuous, positive, and decreasing on $[1, \infty]$ and if $a_n = f(n)$, then

$\sum_{n=1}^{\infty} a_n$ converges if $\int_1^{\infty} f(x)dx$ converges, and

$\sum_{n=1}^{\infty} a_n$ diverges if $\int_1^{\infty} f(x)dx$ diverges.

COMPARISON TESTS

Suppose that $\sum_{n=1}^{\infty} a_n$ and are two series of positive terms and the convergence of $\sum_{n=1}^{\infty} b_n$ is known.

Basic Comparison Test

(1) If there exists a number N such that $a_n < b_n$ for n > N, and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

(2) If there exists a number N such that $a_n > b_n$ for n>N, and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

Limit Comparison Test

If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is a non-zero finite number, then $\sum_{n=1}^{\infty} a_n$ converges if $\sum_{n=1}^{\infty} b_n$ converges, and $\sum_{n=1}^{\infty} a_n$ diverges if $\sum_{n=1}^{\infty} b_n$ diverges.

RATIO TEST

(1) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent and therefore convergent.

(2) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or if, $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, the series $\sum_{n=1}^{\infty} a_n$ is divergent

(3) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, this test has no conclusion and another test must be used.

ALTERNATING SERIES TEST

If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ *or* $\sum_{n=1}^{\infty} (-1)^n b_n$ satisfies

(1) $\lim_{n\to\infty} b_n = 0$ and (2) $b_{n+1} \le b_n$ for all n beyond some finite number N,

then the series is convergent.

ALTERNATING SERIES ESTIMATION THEOREM

If S is the sum of an alternating series that satisfies conditions (1) and (2) in the Alternating Series Test, then the size of the error in using the first n terms to estimate S is less than the first terms to be omitted, that is, $|R_n| = |S - S_n| \le b_{n+1}$

NOTES:

(1) The convergence or divergence of a series is not affected if each term is multiplied by a non-zero finite constant.

(2) The sum of two convergent series converges. The sum of a convergent and a divergent series diverges.

(3) In general, a series “behaves like” the series which is found by considering the dominant term in the numerator and the dominant term in the denominator.