MA 182 The Error in Using Numerical Integration
Section 5.9

1. For the definite integral $\int_1^3 e^{-x^2}\,dx$

(a) Use the program NUMINT on your calculator to approximate the value of this integral with n = 20 and write your answers below.

M(20) = T(20) =

(b) If $|E_M|$ and $|E_T|$ are the errors in the Midpoint and Trapezoidal rules, then $|E_M| \le \frac{K(b-a)^3}{24n^2}$ and $|E_T| \le \frac{K(b-a)^3}{12n^2}$, where $|f''(x)| \le K$ for $a \le x \le b$, that is, K is the maximum value of $|f''(x)|$ for $a \le x \le b$.

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To determine the accuracy of the answers for M(20) and T(20), we need to know the maximum value of the absolute value of the second derivative of e^{-x^2} on [1,3].

To find this value ***graphically,***

(1) Set Y1 = e^(-x^2)

(2) **TI-82/83/83+:** Set Y2 = nDeriv(Y1,x,x) and y3 = abs(nDeriv(Y2,x,x))
Deselect Y1 and Y2

TI-86: y2 = abs(der2(y1,x,x)); Deselect y1

(3) Set your graphing window to [1,3] for x and choose an appropriate interval for y and graph. You may have to adjust the y-values if your original guess does not give you a good picture.

(4) The function you should see is the absolute value of the 2nd derivative of Y1. Use the CALC menu of your calculator to determine the <u>largest</u> y-value of this function. If you round the answer at all, round it <u>up.</u> This is the value of K in the formulas for $|E_M|$ and $|E_T|$.
K =

(c) Now use the formulas for $|E_M|$ and $|E_T|$ to determine the maximum value of the error for M(20) and T(20).

(d) Suppose you wanted your answer to have an error < .0000001. Determine the value of n needed to guarantee this accuracy for the Midpoint Rule and for the Trapezoidal Rule.

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2. Most of the time, the greatest accuracy can be achieved using Simpson's Rule. Usually, the simplest way to obtain accuracy for up to as many decimal places as your calculator will show is to start with an arbitrary N value and to keep doubling N until the answers for the program S2N no longer differ in the desired number of decimal places. Use this procedure to obtain an answer with 8-digit accuracy for the integral given in problem #1 above. You can start with N = 20.

3. The following table shows the speedometer readings of a truck, taken at 10-minute intervals, during one hour of a trip.

time (min)	0	10	20	30	40	50	60
speed (mi/h)	40	45	50	60	70	65	60

Determine the distance which the truck traveled during the one hour period using

(a) the Midpoint Rule with n = 3

(b) the Trapezoidal Rule with n = 6

Use the formula

$$T_n = \frac{(b-a)}{2n}[f(x_{o)} + 2f(x_1) + 2f(x_{2)}) + 2f(x_3) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)]$$

(c) Simpson's Rule with n = 6

Use the formula

$$S_n = \frac{(b-a)}{3n}[f(x_{o)} + 4f(x_1) + 2f(x_{2)}) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_{n)})]$$