MA 182 An Application of Differential Equations: Salt in a Reservoir (revised)
Section 7.3

This problem is concerned with a city's water reservoir, fed partly by clean water from a spring and partly by run-off from the surrounding land. In a climate with snow in the winter, the run-off will contain salt which has been put on the roads to make them safe for driving. Our problem will be to determine the concentration of salt in the reservoir.

Suppose the water reservoir holds 100 million gallons of water and supplies a city with 1 million gallons a day. The reservoir is partly refilled by a spring which provides 0.9 million gallons a day, and the rest of the water, 0.1 million gallons a day, comes from run-off from the surrounding land. The spring water is clean, but the run-off contains salt with a concentration of 0.0001 pounds per gallon. Assume there is no salt in the reservoir initially and that the reservoir is well-mixed (i.e. the water taken out each day contains the concentration of salt in the tank at that instant.)

At any time, the concentration of salt is equal to the *quantity* of salt divided by the *volume* of water.

That is, if C is the concentration of salt in the reservoir (in pounds/gallon) and Q is the quantity of salt (in pounds) then $C = \dfrac{Q\,lbs}{100\ million\,gallons} = \dfrac{Q}{100\ million} lbs/gal$.

We will set up a differential equation based on :

Rate of change of the quantity of salt = Rate of salt entering - Rate of salt leaving

First, we need to find the rate at which salt is entering. This is entirely through the run-off, which is 0.1 million gallons per day, with each gallon containing 0.0001 pounds of salt. Therefore

Rate salt entering = Concentration of salt entering X Volume of water entering per day
= 0.0001 (lb/gal) X 0.1 (million gal/day)
= 0.00001 (million lb/day) = 10 (lb/day)

Salt is leaving the reservoir each day due to the water used by the city, so

Rate salt leaving = Concentration of salt in the reservoir X Volume leaving per day

$$= \frac{Q}{100\ million}(lbs/gal)\cdot 1(million\ \ gal/day)$$

$$= \frac{Q}{100}\ (lb/day)$$

Therefore Q must satisfy the differential equation $\dfrac{dQ}{dt} = 10 - \dfrac{Q}{100}$

OVER ⟶

To determine the concentration of salt in the reservoir at any time,

1. Solve the differential equation for Q at time t, subject to the initial condition Q(0) = 0 (since we assumed no salt in the reservoir initially). (Hint: First multiply by 100 and then separate the variables.)

2. Check your result by displaying the solution to the differential equation graphically using EULERGPH on your calculator and then graphing your solution for Q on the same screen.

3. Find a formula for the concentration of salt at time t by using $C = \dfrac{Q}{100\ million}$.

4. Sketch a graph of concentration against time on your calculator.

5. What happens to C for large values of t? What does this mean about the value of C after a long period of time?